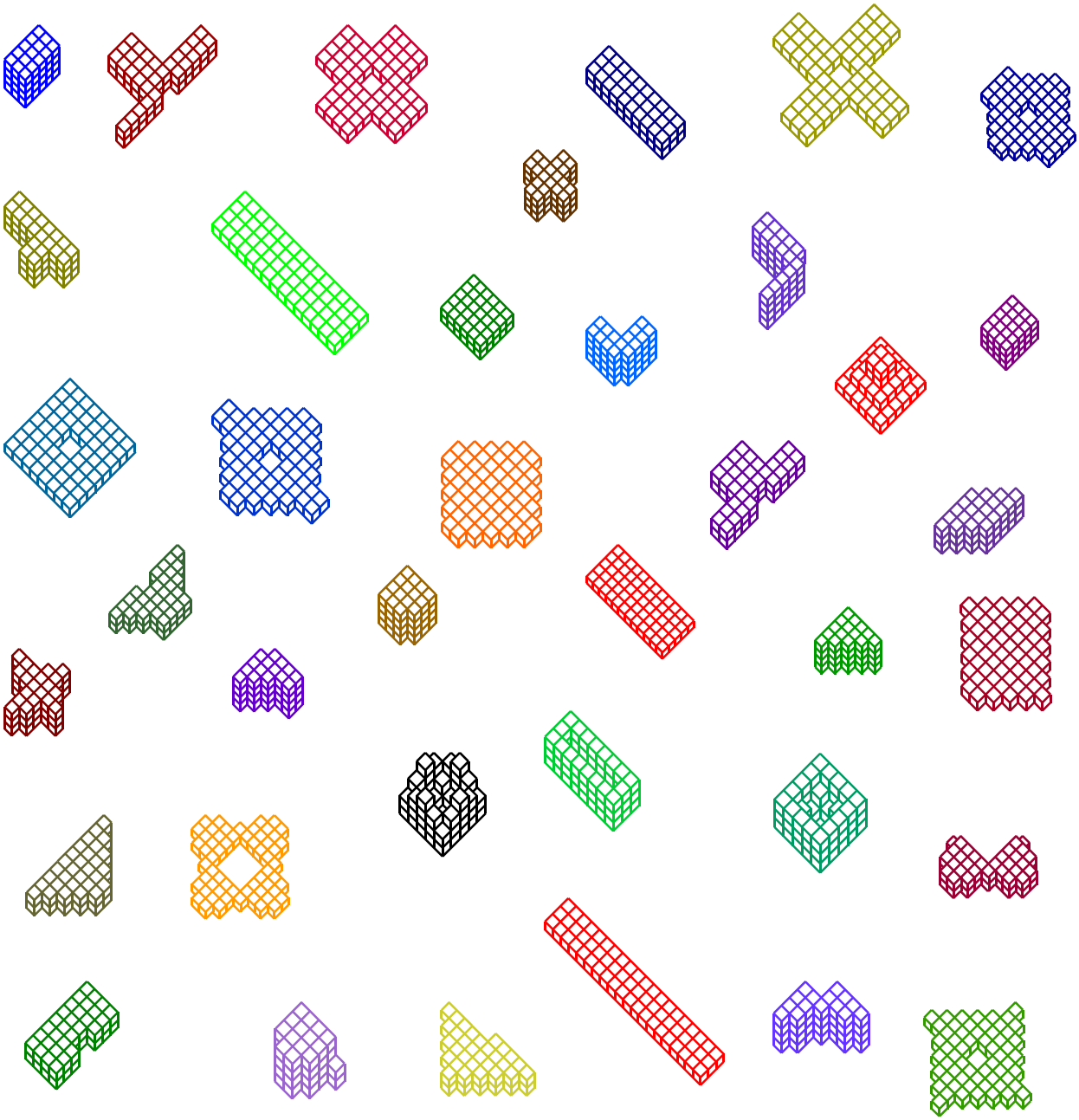
Lesson Three
Using Equations



Class Setting:

By continuing to have them pretend they are all computer engineers working on a research program, these exercises will help them to understand the physical qualities of the computer chips. To do this they will need to learn how to use some of the most basic equations used in science and engineering. In using these equations, they will learn how to calculate the volume, area, surface area and parameter of the pieces. Once they understand these concepts as they apply to the pieces, the older or more advanced students can apply them to different shapes created from the pieces in Lesson Two. Regardless of how advanced they get in their calculations, accuracy and good records are very important. As is typical of most mathematical exercises one step builds on the next and a mistake or sloppy records could cause extra effort or a wrong answer when using the data derived in this lesson in the next lesson plan. Again, the JENZAC™ puzzles pieces will represent the computer chips and the solution grids will serve as the motherboard.

Overview of Using Equations:

Students should know that all science and engineering is typically recorded and/or predicted **Using** a wide verity of **Equations**. Again, they are probably already evaluating and analyzing things using equations, but don't realize it. This exercise is intended to introduce them to the basic concepts and terminology used in equations and how they can be used to understanding the things around us. It will give them a deeper understanding of the JENZAC™ puzzles pieces and help them begin to understand how to expand that knowledge to things they encounter every day.

This lesson may be simple for some and completely new for others. The student's ability to do the more complex exercises will greatly depend on their skill with basic math functions such as addition, subtraction, multiplication and division (we will also introduce square, square rout, and exponents). You may use calculators if you like. Even though they may be just developing the math skills, they can count the squares and cubes to understand the concept of what we will be doing. With this exercise, the most important thing is that we want to convey the concept and understanding of each equation to the students. The students may vary in their ability to extend their understanding of volume, area, surface area, and parameter to the more complex puzzle pieces and/or some of the shapes they create with them; but, it is very important that they understand the concept and how it applies to the pieces and shapes they do work with.

Also, after they have made a good honest attempt, it's OK to give them the solutions to a few of the more complex shapes. By having the solution they will see how the pieces interlock and be encouraged to try to solve some of the other shapes and spend more time understanding the equations they will be studding in these lessons.

An **equation** is a mathematical statement, in symbols, that two things are the same (or equivalent). Equations are written with an equal sign, as in:

$6 + 2 = 8$ or $6 - 2 = 4$ or $6 \times 2 = 12$ or $6 \div 2 = 3$

We want to use equations to measure, understand or predict things about the object we're observing.

> ***Our techniques will be to use simple equations along with counting squares and cubes to understand physical properties (i.e. volume, area, surface area, and parameter) of the JENZAC™ puzzle pieces and shapes. Here we want the students to really understand what these terms mean and how to calculate them.***

Another very important precept for students to learn is to capitalize on information they have already developed. If they have kept good records of the different solutions during Lesson Two, they can use that information in Lesson Three. And the information they will develop in this lesson will be used in Lesson Four. Also, by having already solved some shapes in Lesson Two, they can try to solve additional ones in this lesson.

List of Materials Needed:

- 1 set of JENZAC™ pieces per group of students
- Ruler (1 per group of students)
- Paper (1 per student)
- Pens/Pencils (if needed)

Activity Time Frame:

- Two one hour time blocks as a minimum. More time should be allowed if the students are able to work with the more complex shapes.

Environmental Setting:

- A classroom with semi-large tables with space enough to build small cubes.

PASS Objectives:

The student will:

- Select the most logical conclusion for given experimental data.
- Communicate scientific procedures and explanations.
- The student will engage in investigations that lead to the discovery of the following concepts:
  - The volume of an object is not altered due to changes in shape.
  - The mass of an object is not altered due to changes in shape.
  - The surface area of an object is altered due to changes in shape.
  - The parameter of an object is altered due to changes in shape.

Project Objectives:

The students will:

- Determine the volume of various shapes to lead to the conclusion that volume does not change if the same JENZAC™ pieces are used to solve a different puzzle shape.
- Determine the plan area or foot print area of a 2-dimentional shape to lead to the conclusion that the area does not change if the same JENZAC™ pieces are used to solve a different 2-dimentional puzzle shape.
- Evaluate the parameter of various shapes to lead to the conclusion that the parameter does change with different shapes if the same JENZAC™ pieces are used to solve a different puzzle shape.
- Evaluate the surface area of various shapes to lead to the conclusion that surface area does change with different shapes if the same JENZAC™ pieces are used to solve a different puzzle shape.

Vocabulary Terms

- Equal Sign
- Plus Sign
- Multiplication Sign
- Division Sign
- Volume
- Surface Area
- Cube
- Prism
- Box
- Mass

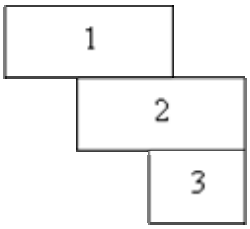
Background Knowledge:

Volume - The volume of a box is given by the equation:

V=l*w*h

- V = the volume of the box
- l = the length of the box
- w = the width of the box
- h = the height of the box

Since several of the JENZAC™ pieces are not perfect boxes, the volume of these individual pieces must be evaluated in parts. For example, with a piece such the one shown below, it can be divided into three separate boxes as shown.



Shape Volumes - The volumes of these three boxes are then added together to get the entire volume of the puzzle piece. Each puzzle piece volume is calculated in the same manner. The volumes of the pieces are then added together to get the entire volume of the shape.

Surface Area - The surface area of box (or rectangular prism) is found by adding the areas of each of the six sides of the box together:

$$SA = A_1+A_2+A_3+A_4+A_5+A_6$$

Area - The area of a square or rectangular is found the following equation:

$$A=a*b$$

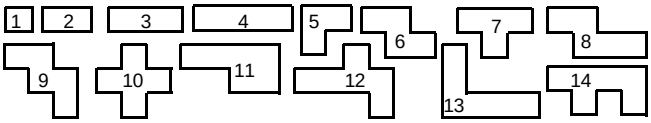
- A = the area
- a = the length of one side
- b = the length of the other side

Surface area for shapes as shown above can be determined in the same method as determining volume, i.e. breaking the shape into several boxes.

Activity Procedures

Session One: Give each team of students a JENZAC™ set. Remind them of their role as computer engineers… Share with the students how equations are used every day… and particularly with scientist and engineers.

- Have the students review the names they assigned to the pieces or computer chips. Make sure the whole class still agrees to the names of each "chip". *(This is intended to be FUN and have lots of energy.)*
- Remind them how to use their Notebooks and that some of the information from the last lesson will be used in this one… and that some in this one will be used in the next… it's good to keep clear records of what you do and check your work with others to assure everyone agrees on what is correct. Give them a moment to check their work with others. *(This is intended to reinforce team work and see how well they have been them following instructions.)*
- If your group has a prior working knowledge of the concepts outlined below in this section or will grasp the concepts of this lesson quickly due to their maturity and experience with equations, you may wish to break the teams into groups. Have each group work with given pieces rather than all 14 pieces. This will save time and allow you to focus on the more complex concepts covered in Session Two. To do this, divide the JENZAC™ set into groups of pieces and assign those pieces to the groups of teams. The following divisions may be used:



  - Groups of 2 – Use pieces (1, 3, 5, 7, 9, 11, 13) and (2, 6, 8, 10, 12, 14)
  - Groups of 3 – Use pieces (1, 4, 7, 10, 13), (2, 5, 8, 11, 14), and (3, 6, 9, 12)
  - Groups of 4 – Use pieces (1, 5, 9, 13), (2, 6, 10, 14), (3, 7, 11) and (4, 8, 12)
  - Groups of 5 – Use pieces (1, 6, 12), (2, 7, 13), (3, 8, 14), (4, 9, 11) and (5, 10)

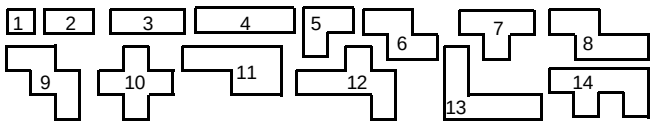
- Explain the concept of Area (Re: Sheet B) and have them calculate the Plan Area or Bottom Area of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheet A). When the students have all the blanks filled in, quickly go through the answers (Re: Sheet M) and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Explain the concept of Side Area (Re: Sheet F) and have them calculate the Side Areas of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheets C, D, E). When the students have all the blanks filled in, quickly go through the answers (Re: Sheet M) and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Explain the concept of Surface Area (Re: Sheet H) and have them calculate the Surface Areas of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheet G). When the students have all the blanks filled in, quickly go through the answers (Re: Sheet M) and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*

- Explain the concept of Parameter (Re: Sheet J) and have them calculate the Parameter of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheet I). When the students have all the blanks filled in, quickly go through the answers (Re: Sheet M) and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Explain the concept of Volume (Re: Sheet L) and have them calculate the Volume of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheet K). When the students have all the blanks filled in, quickly go through the answers (Re: Sheet M) and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Explain the importance of keeping clear organized records of your findings. Have them summarize their findings on one sheet (Re: Sheet N). Before showing them the summary sheet, you might want them to brainstorm what a summery sheet would look like and see how they would structure the information and use that to help them use the summary sheet provided. *(This is intended be interactive, help them organize their thoughts, and check their work.)*
- Review the vocabulary words introduced in this lesson. *(This is intended to make sure everyone is on the same page and focused on using and understanding equations.)*

Session Two: Give each team of students a **JENZAC™** set. Review the equations they used in Session One. Now it's time to see how the equations are used with different shapes that are created using the **JENZAC™** pieces shown on the respective work sheets. Here also, they will be discover if the area, parameter, surface area, or volume change even though the same **JENZAC™** pieces are used to create a different shape.

- Have the students review the names they assigned to the pieces or computer chips. Have the students review and share solutions to any puzzle shapes they have solved in the past. Note the organization skills of the best teams as goals for the rest and discuss the value of good records. *(This is intended to be FUN and have lots of energy.)*
- Remind them how to use their Notebooks and that some of the information from the last lesson will be used in this one… and that some in this one will be used in the next… it's good to keep clear records of what you do and check your work with others to assure everyone agrees on what is correct. Give them a moment to check their work with others. *(This is intended to reinforce team work and see how well they have been them following instructions.)*
- If time allows, it would be good for all students to work through all the work sheets for this session. However; based on your observation of the teams, you may see some teams able to solve more complex shapes than others and time my be too short to allow everyone to work them all. If so, to save time or boredom, you may want to distribute the work sheets for this session to different teams based on their ability to solve the shapes.

  The first shape the team works with using the equations should be one they have solved before. (Remember, the solutions made in Lesson Two can be used in this lesson.) It is good for the students to use their past solutions or even solutions you can provide from the teacher's help sheets. The assembling of different shapes using a provided solution lets them focus on the equations they will be solving rather than the task of solving the shape. After they begin to understand the application of the equations to the shapes, they can solve new shapes and apply the equations to them. To do this, use the following pieces from the **JENZAC™** set and assign those pieces to the respective teams. The following divisions may be used:



2 - Dimensional Shapes

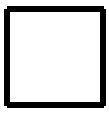
- Easy – Use pieces (1, 3, 5, 7, 14) and (Sheet O with two shapes)
- Moderate – Use pieces (1, 2, 3, 4, 6, 9, 13) and (Sheet Q with two shapes)
- *Moderate/Hard – Use pieces (1, 2, 3, 5, 6, 7, 9, 10) and (Sheet Y with three shapes) (Note: These pieces are the same ones used to solve the 3 – Dimensional Shapes Below)*
- Hard – Use pieces (1, 2, 3, 4, 6, 9, 10, 13, 14) and (Sheet S with two shapes)
- Difficult – Use pieces (1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 13, 14) and (Sheet U with two shapes)

3 - Dimensional Shapes

- Easy – Use pieces (1, 3, 5, 6, 7, 9, 10) and (Sheet W with a 3-D cube shape)
- Moderate – Use pieces (1, 3, 5, 6, 7, 9, 10) and (Sheet AA with one 2-D and 3-D shape)
- Hard – Use pieces (1, 3, 5, 6, 7, 9, 10) and (Sheet CC with one 3-D shape)

- Review the concept of Area (Re: Sheet B) and have them calculate the Plan Area or Bottom Area for their respective shape. You may want to pick a shape and solve it on the board and then leave the remainder for them to solve with their respective teams. (Re: Sheet P, R, T, V, X, Z, BB, DD). When the students have all the blanks filled in, quickly go through the answers and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Review the concept of Side Area (Re: Sheet F) and have them calculate the Side Areas of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheet P, R, T, V, X, Z, BB, DD). When the students have all the blanks filled in, quickly go through the answers and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Review the concept of Surface Area (Re: Sheet H) and have them calculate the Surface Areas of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheet P, R, T, V, X, Z, BB, DD). When the students have all the blanks filled in, quickly go through the answers and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Review the concept of Parameter (Re: Sheet J) and have them calculate the Parameter of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheet P, R, T, V, X, Z, BB, DD). When the students have all the blanks filled in, quickly go through the answers and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Review the concept of Volume (Re: Sheet L) and have them calculate the Volume of each piece. You may want to pick a few pieces and solve them on the board and then leave the remainder for them to solve as a team. (Re: Sheet P, R, T, V, X, Z, BB, DD). When the students have all the blanks filled in, quickly go through the answers and have them make any corrections as necessary. Spend time as needed to assure they understand the concept. *(This is intended to provide more instruction with them doing what you do.)*
- Review the importance of keeping clear organized records of your findings. Have them discuss any observations they have discovered. Review the **Project Objectives** outlined above and guide the class to the appropriate conclusions. *(This is intended be interactive, help them understand the concepts of what they have been doing.)*
- Review the vocabulary words introduced in this lesson. *(This is intended to make sure everyone is on the same page and focused on using and understanding equations.)*

Calculate the **Plan Area (or bottom area)** as you look down at each piece below:



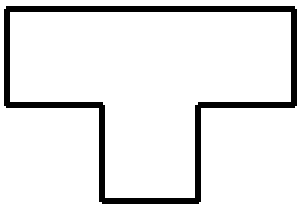
A = _____

The equation for the **Area** of a square is:

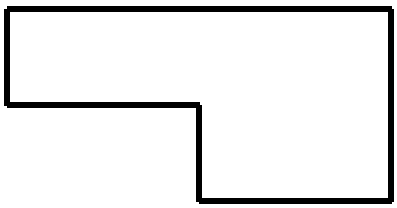
Area = Length x Width

- Or -

A = L x W

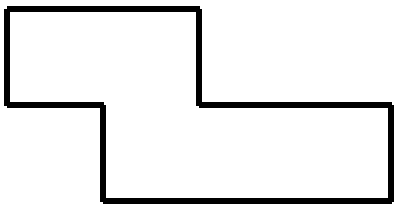


A = _____



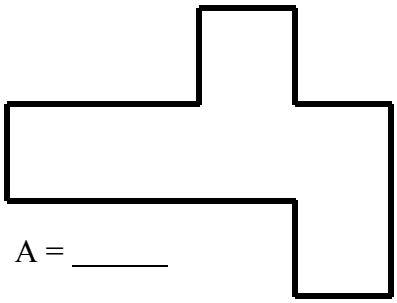
A = _____

A = _____



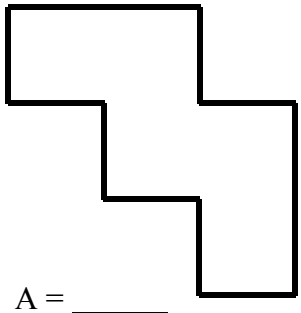
A = _____

A = _____

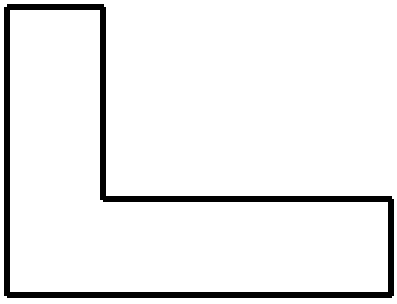


A = _____

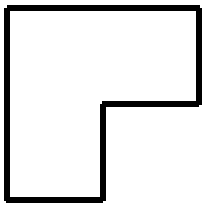
A = _____



A = _____



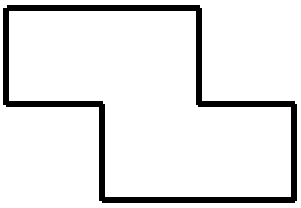
A = _____



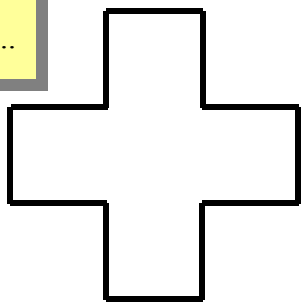
A = _____

You can also break an object into smaller areas, then add them up to get to total area:

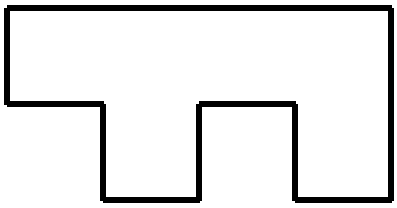
Area = $A_1 + A_2 + A_3$ ...



A = _____

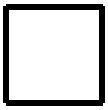


A = _____



A = _____

Calculate the **Plan Area (or footprint)** as you look down at each piece below:



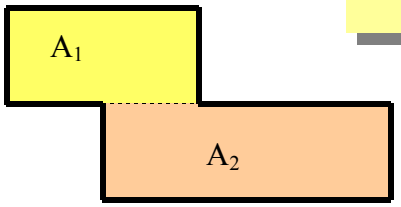
A = 1

The equation for the **Area** of a square is:
Area = Length x Width
- Or -
A = L x W

A = L x W
- Or -
A = 1 x 1 = 1

You can also break an object into smaller areas, then add them up to get to total area:
Area = $A_1 + A_2 + A_3$ ...

With any square area
A = L x W
And the total area is:
Area = $A_1 + A_2 + A_3$ ...
- With -
$A_1 = 2 \times 1 = 2$
$A_2 = 3 \times 1 = 3$
- Then -
$A_{Bottom} = A_1 + A_2 = 5$

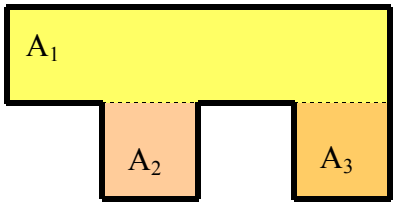


A = 5

Have them discover the area is the same no matter how you flip or rotate the piece as long is it is laying flat.

Or (to save time) count the squares:
(5 for the piece above and 6 for the piece below)

Break the equations into small pieces for them to understand. Take time to explain the variables (or letters) and how their subscripts work. Remind them, the subscripts are just there to keep the different areas separate in the equation, they have no numerical impact on the calculations.



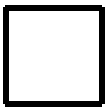
A = 6

Or, with any square area
A = L x H
And the total area is:
Area = $A_1 + A_2 + A_3$ ...
- With -
$A_1 = 4 \times 1 = 4$
$A_2 = 1 \times 1 = 1$
$A_3 = 1 \times 1 = 1$
- Then -
$A = A_1 + A_2 + A_3 = 6$

Calculate the **Side Areas** with the pieces positioned as shown

Back side
A_B = _____

Left side
A_L = _____



Right side
A_R = _____

Front side
A_F = _____

Back side
A_B = _____

Left side
A_L = _____

Right side
A_R = _____

Front side
A_F = _____

The **Side Area** is the sum of all the areas on that given side:

Left Side Area = $A_L = A_{L1} + A_{L2} + A_{L3} \ldots$

- And -

Back Side Area = $A_B = A_{B1} + A_{B2} + A_{B3} \ldots$

- So -

Using similar letters and subscripts, write an equation for the Front and Right side areas.

Back side
A_B = _____

Left side
A_L = _____

Right side
A_R = _____

Front side
A_F = _____

Back side
A_B = _____

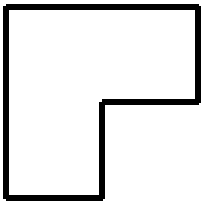
Left side
A_L = _____

Right side
A_R = _____

Front side
A_F = _____

Back side
A_B = _____

Left side
A_L = _____

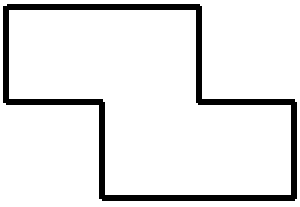


Right side
A_R = _____

Front side
A_F = _____

Back side
A_B = _____

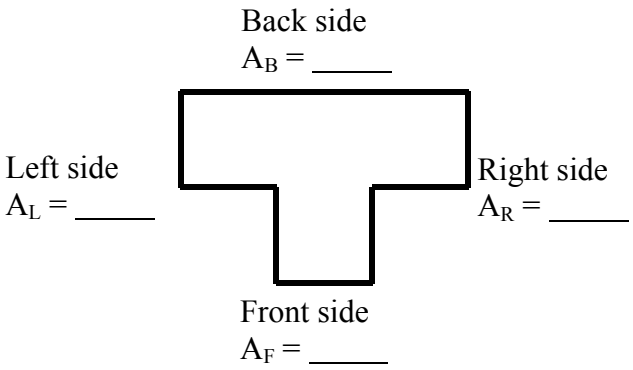
Left side
A_L = _____

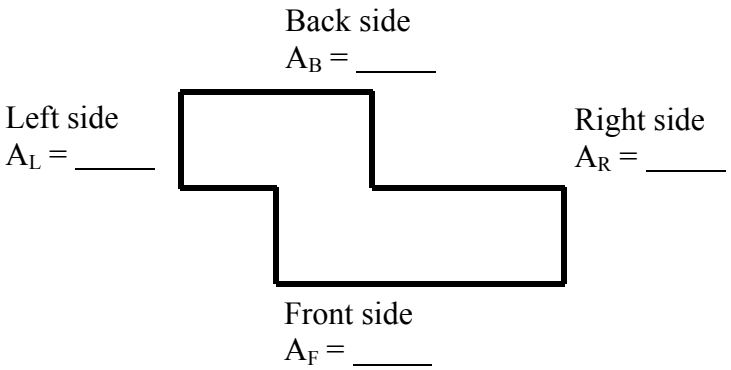


Right side
A_R = _____

Front side
A_F = _____

Calculate the **Side Areas** with the pieces positioned as shown below:





The **Side Area** is the sum of all the areas on that given side:

Left Side Area = $A_L = A_{L1} + A_{L2} + A_{L3} \ldots$

- And -

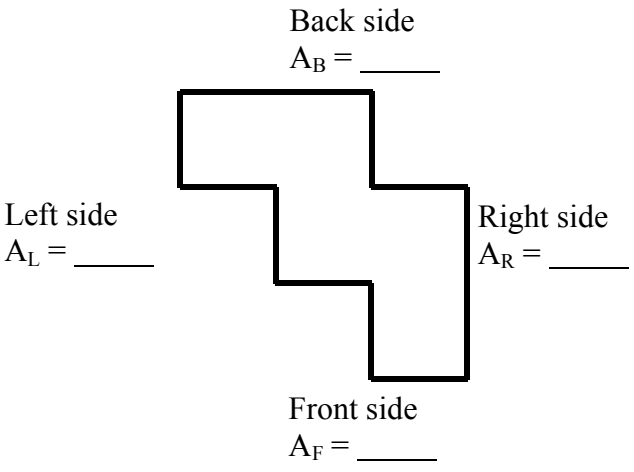
Back Side Area = $A_B = A_{B1} + A_{B2} + A_{B3} \ldots$

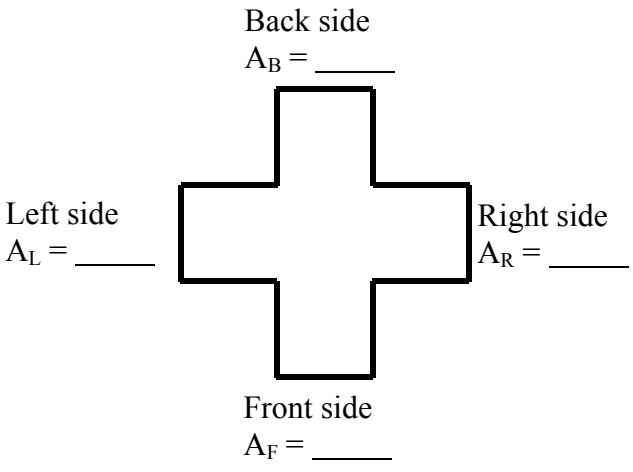
- And -

Right Side Area = $A_R = A_{R1} + A_{R2} + A_{R3} \ldots$

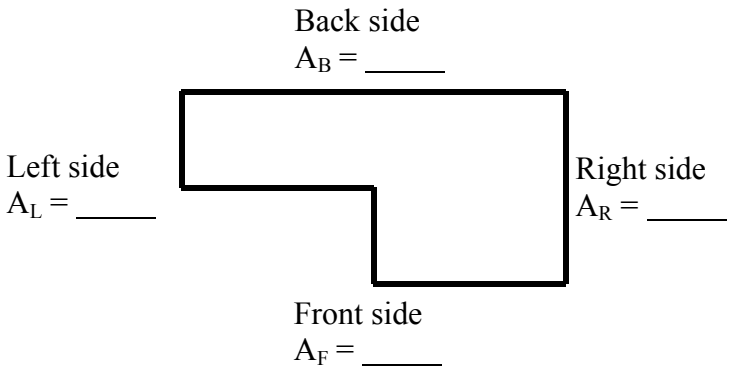
- And -

Front Side Area = $A_F = A_{F1} + A_{F2} + A_{F3} \ldots$





Calculate the **Side Areas** with the pieces positioned as shown



The **Side Area** is the sum of all the areas on that given side:

Left Side Area = $A_L = A_{L1} + A_{L2} + A_{L3} \ldots$

- And -

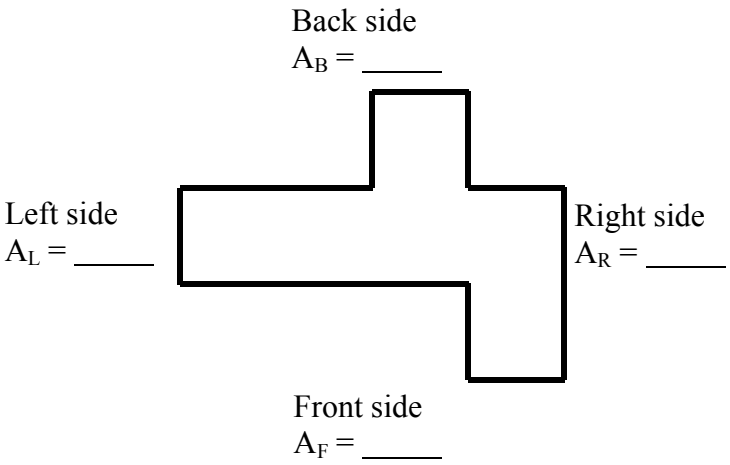
Back Side Area = $A_B = A_{B1} + A_{B2} + A_{B3} \ldots$

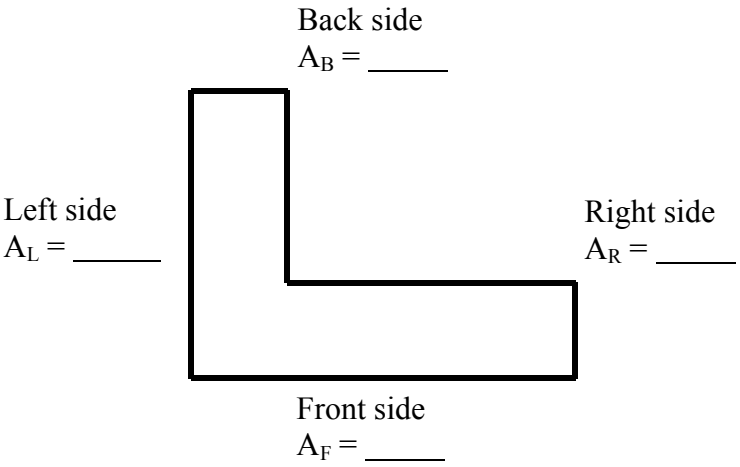
- And -

Right Side Area = $A_R = A_{R1} + A_{R2} + A_{R3} \ldots$

- And -

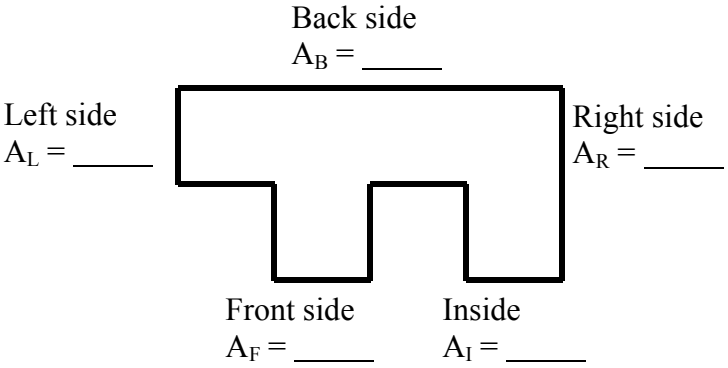
Front Side Area = $A_F = A_{F1} + A_{F2} + A_{F3} \ldots$



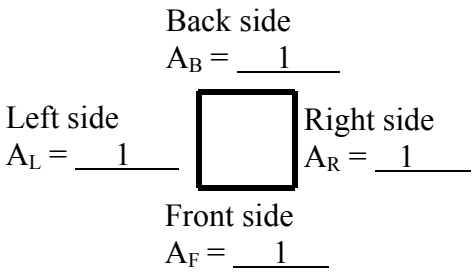


The **Inside Area** is the sum of all the areas you can't see looking from the top, bottom, front, back left, or right side:

Inside Side Area = $A_I = A_{I1} + A_{I2} + A_{I3} \ldots$

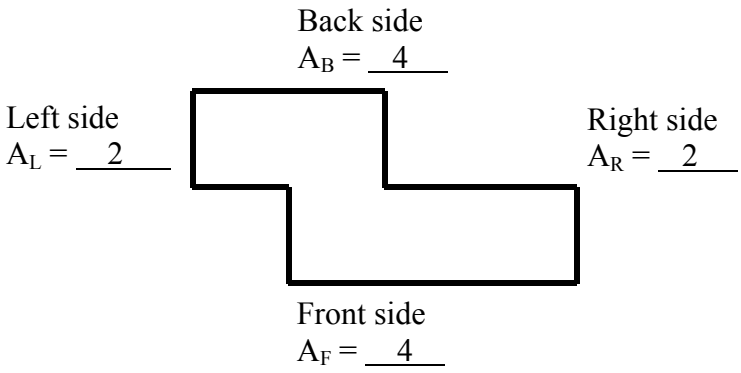


Calculate the **Side Areas** with the pieces positioned as shown



The equation for the **Area** of a square is:
Area = Length x Height
- Or -
A = L x H
- So -
The **Side Area** is the sum of all the areas on that given side:
Left Side Area = $A_L = A_{L1} + A_{L2} + A_{L3} \ldots$
- And -
Back Side Area = $A_B = A_{B1} + A_{B2} + A_{B3} \ldots$
- And -
Right Side Area = $A_R = A_{R1} + A_{R2} + A_{R3} \ldots$
- And -
Front Side Area = $A_F = A_{F1} + A_{F2} + A_{F3} \ldots$

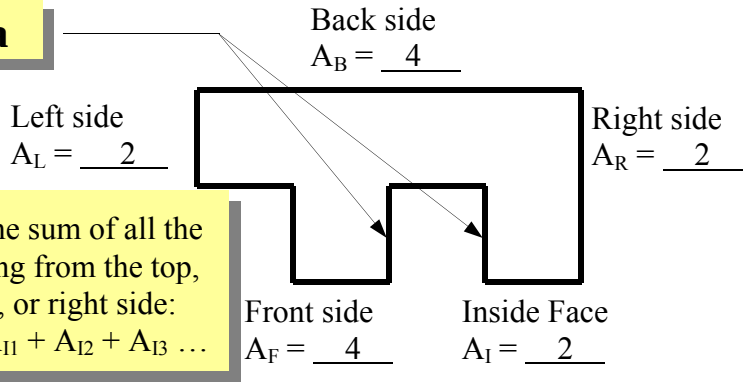
With any square area
A = L x H
The Side Areas are:
$A_L = A_{L1} + A_{L2} + A_{L3} \ldots$
$A_B = A_{B1} + A_{B2} + A_{B3} \ldots$
$A_R = A_{R1} + A_{R2} + A_{R3} \ldots$
$A_F = A_{F1} + A_{F2} + A_{F3} \ldots$
- Then -
$A_L = 1 + 1$
$A_B = 2 + 2$
$A_R = 1 + 1$
$A_F = 1 + 3$



Remember (to save time) they can count the squares on the sides, like they did with the bottom area, to get the side areas, it's just important that they understand how to calculate them as well.

Note here that the subscripts in the equations now are both letters and numbers. Explain how the numbers are just a way to show in the equation another area on that side of the piece. Have them note the … in the equation. Explain how that this just indicates the pattern will continue for as many areas as they have on that particular side.

Inside Area



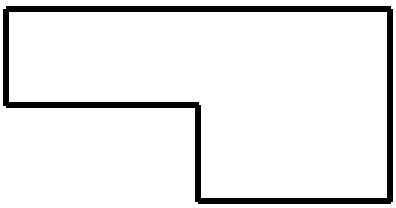
The **Inside Area** is the sum of all the areas you can't see looking from the top, bottom, front, back left, or right side:
Inside Side Area = $A_I = A_{I1} + A_{I2} + A_{I3} \ldots$

With any square area
A = L x H
The Side Areas are:
$A_L = A_{L1} + A_{L2} + A_{L3} \ldots$
$A_B = A_{B1} + A_{B2} + A_{B3} \ldots$
$A_R = A_{R1} + A_{R2} + A_{R3} \ldots$
$A_F = A_{F1} + A_{F2} + A_{F3} \ldots$
$A_I = A_{I1} + A_{I2} + A_{I3} \ldots$
- Then -
$A_L = 1 + 1$
$A_B = 4$
$A_R = 2$
$A_F = 1 + 1 + 1 + 1$
$A_{IF} = 1 + 1$

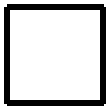
Calculate the **Surface Area** for each piece below:

The equation for the **Surface Area** of a shape is:
Surface Area = sum of all Side Areas

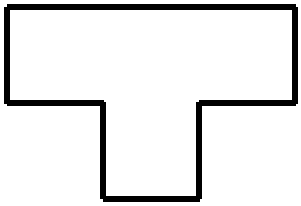
(Remember the
bottom and top areas are side areas also.)



SA = _____



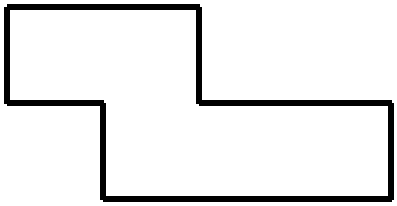
SA = _____



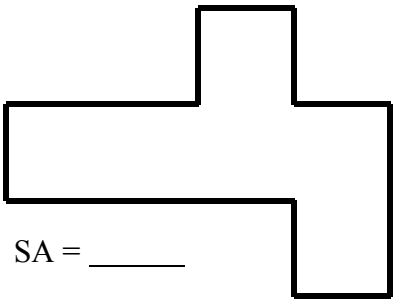
SA = _____

- Or, another way to say that is -
$SF = \sum A_X$ (where X is any side)

SA = _____



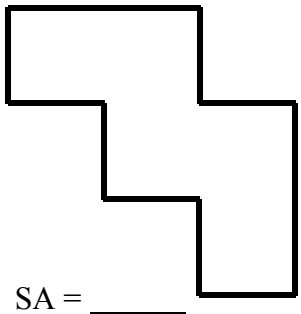
SA = _____



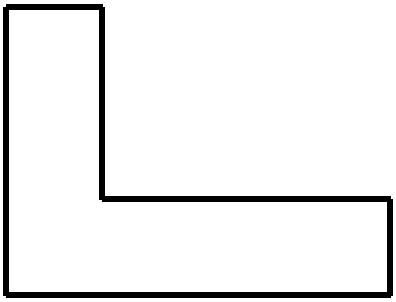
SA = _____

SA = _____

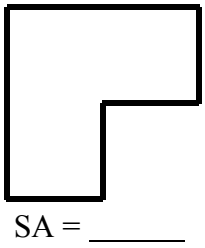
SA = _____



SA = _____

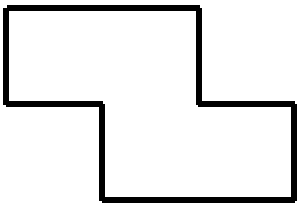


SA = _____

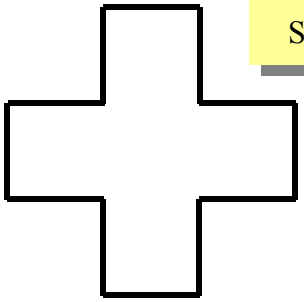


SA = _____

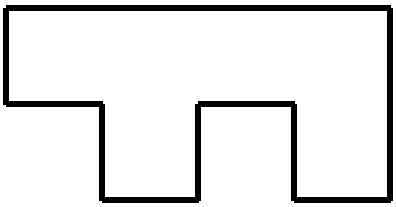
- Or, another way to write that is -
$SF = A_L + A_B + A_R + A_F + A_{Top} + A_{Bottom} \ldots$



SA = _____

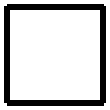


SA = _____



SA = _____

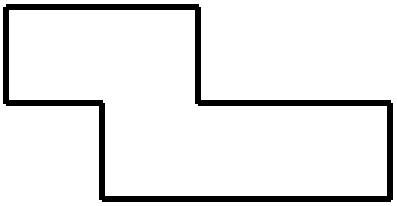
Calculate the **Surface Area** for each piece below:



SA = 6

The equation for the **Surface Area** of a shape is:
Surface Area = sum of all Side Areas
- Or -
$SF = \sum A_X$ (where X is any side)
- Or -
$SF = A_L + A_B + A_R + A_F + A_{Top} + A_{Bottom} \ldots$

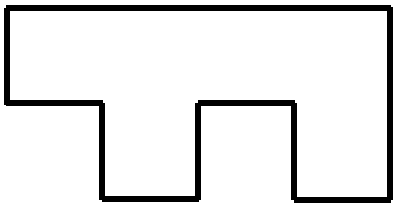
The Side Areas were:
$A_L = A_{L1} + A_{L2} + A_{L3} \ldots$
$A_B = A_{B1} + A_{B2} + A_{B3} \ldots$
$A_R = A_{R1} + A_{R2} + A_{R3} \ldots$
$A_F = A_{F1} + A_{F2} + A_{F3} \ldots$
- So -
$A_L = 1 + 1 = 2$
$A_B = 2 + 2 = 4$
$A_R = 1 + 1 = 2$
$A_F = 1 + 3 = 4$
$A_{Top} = A_{Bottom} = 5$
- Then -
$SF = A_L + A_B + A_R + A_F \ldots$
- Or -
$SF = 2 + 4 + 2 + 4 + 5 + 5 = 22$
- Or -
$SF = 2 + 4 + 2 + 4 + (2 \times 5) = 22$



SA = 22

It's very important for the students to see the equations. They may seem intimidating at first; but, as they study them they will see they represent calculations they can do in their heads. **Equations are just a way to write down what we're thinking.**

Note that you can just add up the side areas or see that the top equals the bottom and use 2 x the bottom area (2 x 5) in your equation vs. 5 + 5

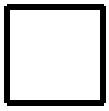


SA = 26

The Side Areas were:
$A_L = A_{L1} + A_{L2} + A_{L3} \ldots$
- And so on, So -
$A_L = 1 + 1 = 2$
$A_B = 4$
$A_R = 2$
$A_F = 1 + 1 + 1 + 1 = 4$
$A_I = 1 + 1 = 2$
$A_{Top} = A_{Bottom} = 6$
- Then -
$SA = A_L + A_B + A_R + A_F \ldots$
- Or -
$SA = 2 + 4 + 2 + 4 + 2 + 6 + 6 = 26$
- Or -
$SA = 2 + 4 + 2 + 4 + 2 + (2 \times 6) = 22$

Calculate the **Parameter** of the footprint for each piece

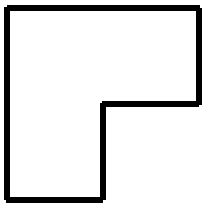
The equation for the **Parameter** of a shape is:
Parameter = Length (1) + Length (2) + Length (3) ...
- Or -
$P = L_1 + L_2 + L_3 ...$

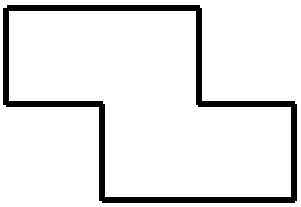

P = _____

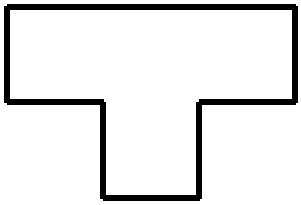

P = _____

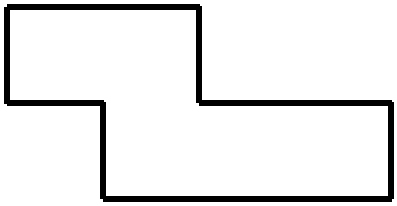

P = _____

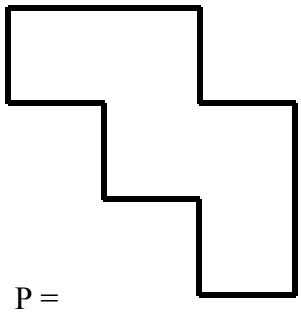

P = _____

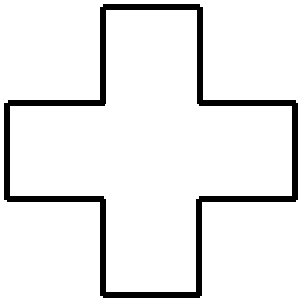

P = _____

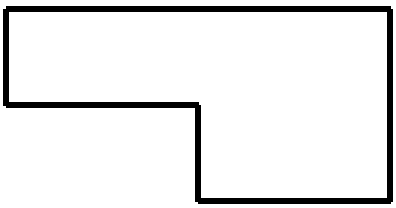

P = _____

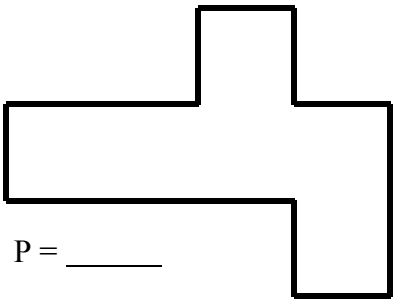

P = _____

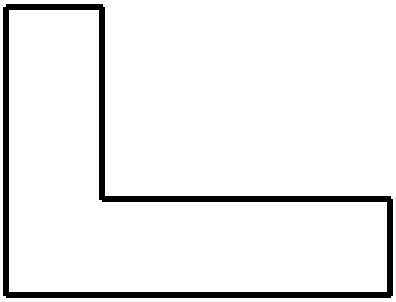

P = _____

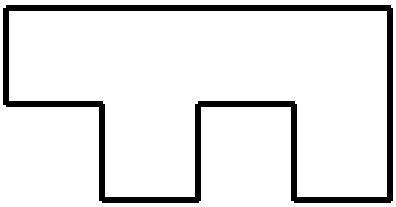

P = _____


P = _____


P = _____

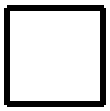

P = _____


P = _____


P = _____

Calculate the **Parameter** of the footprint for each piece

The equation for the **Parameter** of a shape is:
Parameter = Length (1) + Length (2) + Length (3) ...
- Or -
$P = L_1 + L_2 + L_3 \ldots$

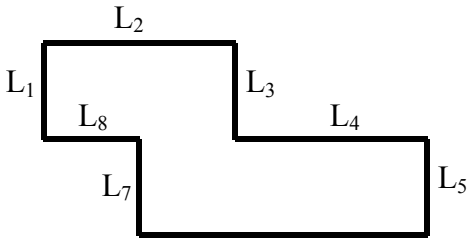


P = 4

$P = L_1 + L_2 + L_3 \ldots$
- Or -
$P = 1 + 1 + 1 + 1 = 4$

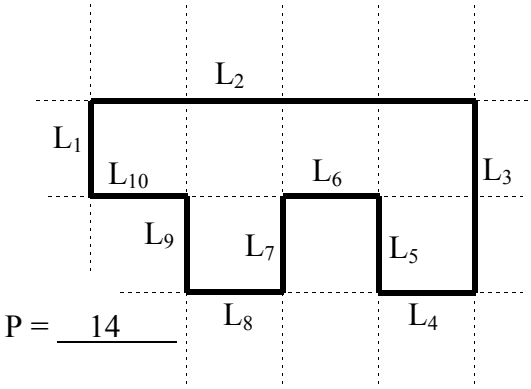
You can also calculate the Parameter of a square with equal sides as:
$P = L \times 4$

$P = L_1 + L_2 + L_3 \ldots$
- Then -
$P = 1 + 2 + 1 + 2 + 1 + 3 + 1 + 1 = 12$



P = 12

Don't forget to use a grid to measure or count the length.

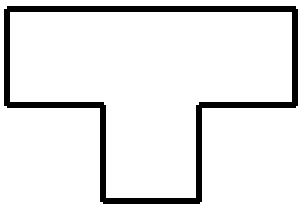


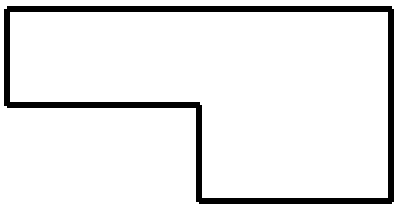
P = 14

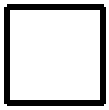
$P = L_1 + L_2 + L_3 \ldots$
- Then -
$P = 1 + 4 + 2 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 14$

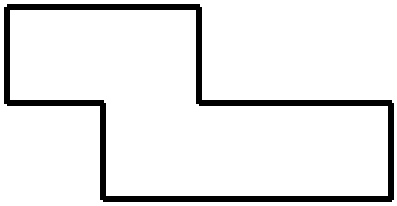
Calculate the **Volume** for each piece below:

The equation for the **Volume** of a box is:
Volume = Length x Width x Height
- Or -
V = L x W x H

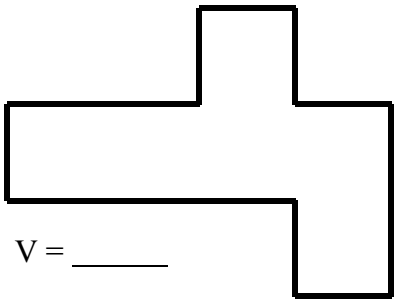

V = _____


V = _____


V = _____

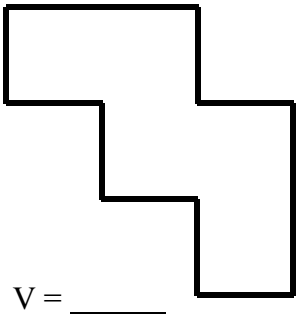

V = _____

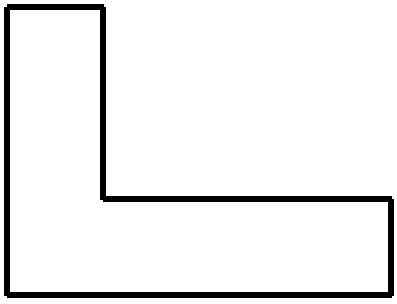

V = _____

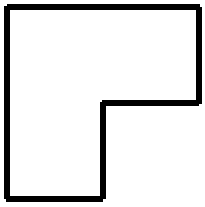

V = _____


V = _____

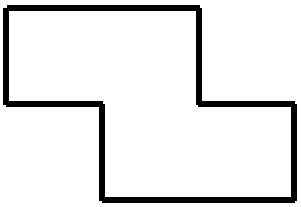

V = _____

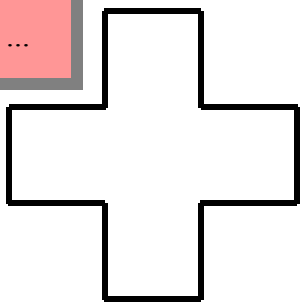

V = _____

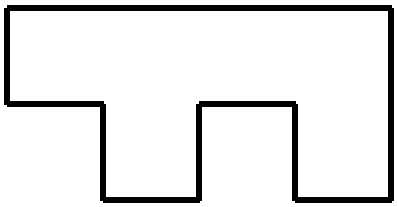

V = _____


V = _____

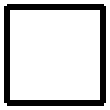
You can also break an object into smaller boxes, then add them up to get to total volume:
Volume = $V_1 + V_2 + V_3$ ...


V = _____


V = _____


V = _____

Calculate the **Volume** for each piece below:



V = 1

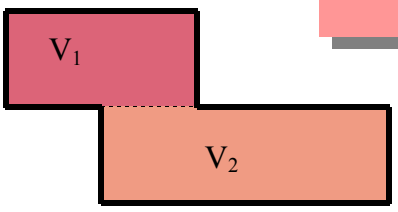
The equation for the **Volume** of a box (which is technically called a rectangular prism) is:
Volume = Length x Width x Height
- Or -
V = L x W x H

V = L x H
- Or -
V = 1 x 1 = 1

Note: with a Cube all sides are equal:
- Or -
S = L = W = H
- So -
$V = S \times S \times S = S^3$

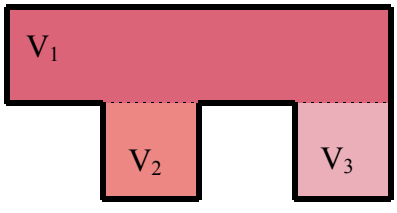
You can also break an object into smaller boxes, then add them up to get to total volume:
Volume = $V_1 + V_2 + V_3$ ...

With any box area
V = L x W x H
And the total volume is:
Volume = $V_1 + V_2 + V_3$ ...
- With -
$V_1 = 2 \times 1 = 2$
$V_2 = 3 \times 1 = 3$
- Then -
$V = V_1 + V_2 = 5$



V = 5

Or count the cubes:
(5 for the piece above and 6 for the piece below)



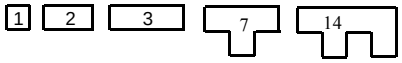
V = 6

Or, with any box
V = L x W x H
And the total volume is:
Volume = $V_1 + V_2 + V_3$ ...
- With -
$V_1 = 4 \times 1 = 4$
$V_2 = 1 \times 1 = 1$
$V_3 = 1 \times 1 = 1$
- Then -
$V = V_1 + V_2 + V_3 = 6$

Piece Number	Piece Shape	No. Cubes	Plan Area	Perimeter	Areas by Location							Surface Area	Volume
					Top	Bottom	Left Side	Right Side	Front	Back	Inside Faces		
1		1	1	4	1	1	1	1	1	1	0	6	1
2		2	2	6	2	2	1	1	2	2	0	10	2
3		3	3	8	3	3	1	1	3	3	0	14	3
4		4	4	10	4	4	1	1	4	4	0	18	4
5		3	3	8	3	3	2	2	2	2	0	14	3
6		4	4	10	4	4	2	2	3	3	0	18	4
7		4	4	10	4	4	2	2	3	3	0	18	4
8		5	5	12	5	5	2	2	4	4	0	22	5
9		5	5	12	5	5	3	3	3	3	0	22	5
10		5	5	12	5	5	3	3	3	3	0	22	5
11		6	6	12	6	6	2	2	4	4	0	24	6
12		6	6	14	6	6	3	3	4	4	0	26	6
13		6	6	14	6	6	3	3	4	4	0	26	6
14		6	6	14	6	6	2	2	4	4	2	26	6

Piece Shape	Piece Number	Length of Primeter	Areas by Location							Surface Area	Volume
			Top	Bottom	Left Side	Right Side	Front	Back	Inside Faces		
	1										
	2										
	3										
	4										
	5										
	6										
	7										
	8										
	9										
	10										
	11										
	12										
	13										
	14										

Use the pieces shown to solve the following two puzzles:
(Note: There is more than one solution.)

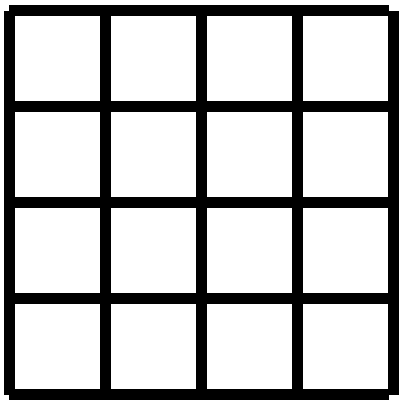


Calculate the **Bottom and Top Area** for the two shapes shown:

Calculate the **Side Areas** of the two shapes shown:

Back side
A_B = _____

Left side
A_L = _____



Right side
A_R = _____

Top Area
A_{Top} = _____

Bottom Area
A_{Bottom} = _____

Front side
A_F = _____

Calculate the **Surface Area** of the two shapes shown:

SA = _____

P = _____

Calculate the **Parameter** of the two shapes shown:

V = _____

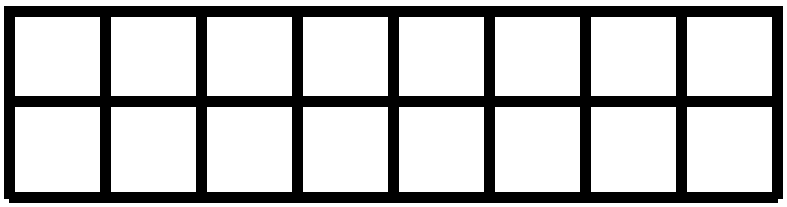
Calculate the **Volume** of the two shapes shown:

Top Area
A_{Top} = _____

Back side
A_B = _____

Bottom Area
A_{Bottom} = _____

Left side
A_L = _____



Right side
A_R = _____

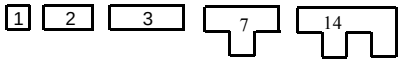
Front side
A_F = _____

SA = _____

V = _____

P = _____

Use the pieces shown to solve the following two puzzles:
(Note: There is more than one solution.)



Calculate the **Bottom and Top Area** for the two shapes shown:

See Teacher Sheets B for equations and comments.

Note: While there is more than one solution to the puzzle there is only one answer to the questions.

Back side
A_B = 4

Left side
A_L = 4

Top Area
A_{Top} = 16

SA = 48

14	14	2	3
14	1	2	3
14	14	7	3
14	7	7	7

Front side
A_F = 4

V = 16

Right side
A_R = 4

Bottom Area
A_{Bottom} = 16

P = 16

Calculate the **Side Areas** with the two shapes shown:

See Teacher Sheets F for equations and comments.

Calculate the **Surface Area** for the two shapes shown:

See Teacher Sheets H for equations and comments.

Calculate the **Volume** for the two shapes shown:

See Teacher Sheets L for equations and comments.

Calculate the **Parameter** of the two shapes shown:

See Teacher Sheets J for equations and comments.

Top Area
A_{Top} = 16

Back side
A_B = 8

Bottom Area
A_{Bottom} = 16

Left side
A_L = 2

2	2	7	7	7	14	1	14
3	3	3	7	14	14	14	14

Right side
A_R = 2

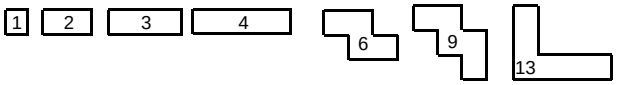
Front side
A_F = 8

SA = 52

V = 16

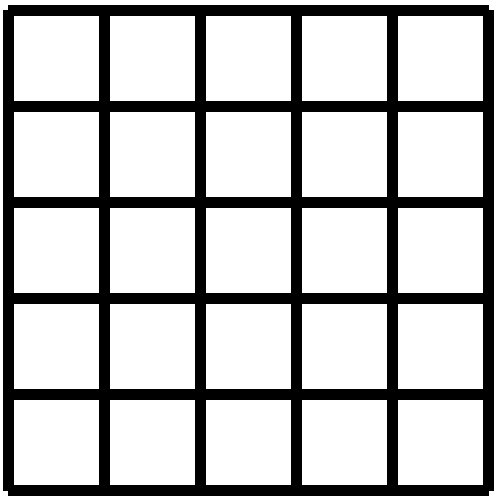
P = 20

Use the pieces shown to solve the following puzzles:
(Note: There is more than one solution.)



Calculate the **Bottom and Top Area** for the two shapes shown:

Back side
A_B = _____



Left side
A_L = _____

Right side
A_R = _____

Top Area
A_{Top} = _____

Bottom Area
A_{Bottom} = _____

Front side
A_F = _____

Calculate the **Surface Area** for the two shapes shown:

SA = _____

Calculate the **Side Areas** with the two shapes shown:

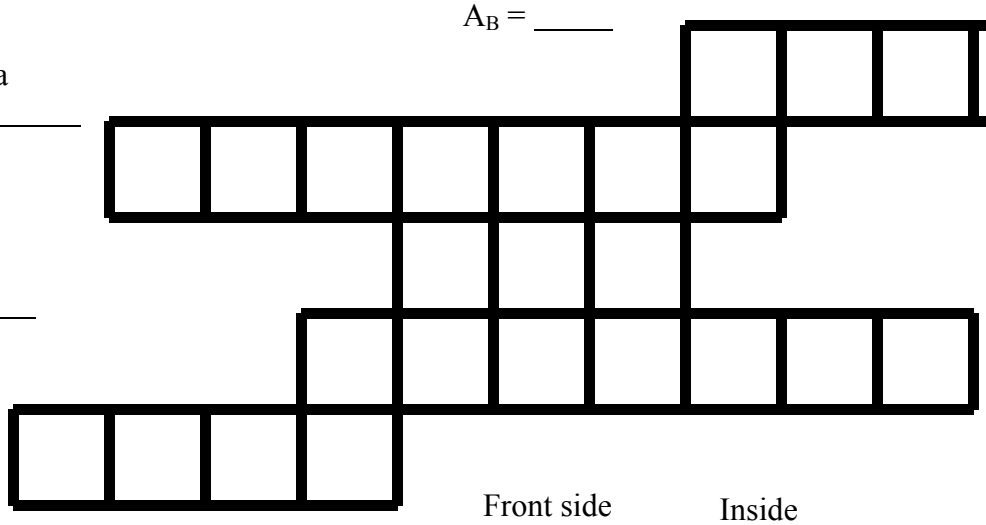
Calculate the **Parameter** of the two shapes shown:

P = _____

Calculate the **Volume** for the two shapes shown:

V = _____

Back side
A_B = _____



Top Area
A_{Top} = _____

Bottom Area
A_{Bottom} = _____

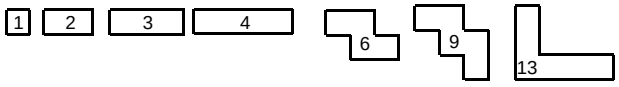
Left side
A_L = _____

Right side
A_R = _____

Front side
A_F = _____

Inside
A_I = _____

SA = _____

Use the pieces shown to solve the following two puzzles: 

(Note: There is more than one solution.)

Calculate the **Bottom and Top Area** for the two shapes shown:
Note the bottom and top areas are the same if the same pieces are used.

Back side
A_B = 5

4	4	4	4	2
13	3	1	6	2
13	3	9	6	6
13	3	9	9	6
13	13	13	9	9

Calculate the **Side Areas** with the two shapes shown:
Note that the side areas are different even though the same pieces are used.

Left side
A_L = 5

Right side
A_R = 5

Top Area
A_{Top} = 25

Bottom Area
A_{Bottom} = 25

Calculate the **Surface Area** for the two shapes shown:
Note that the more compact the shape is the smaller the surface area.

SA = 70

Front side
A_F = 5

P = 20

Calculate the **Parameter** of the two shapes shown:
Note that the parameter is different between the shapes.

Calculate the **Volume** for the two shapes shown:
Note that the volume didn't change

V = 25

Back side
A_B = 11

Top Area
A_{Top} = 25

Bottom Area
A_{Bottom} = 25

							6	3	3	3	
1	13	13	13	13	6	6					
				9	13	6					
			9	9	13	4	4	4	4		
2	2	9	9								

Left side
A_L = 5

Right side
A_R = 5

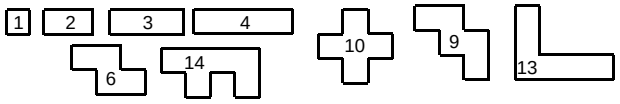
SA = 82

Front side
A_F = 11

Inside
A_I = 12

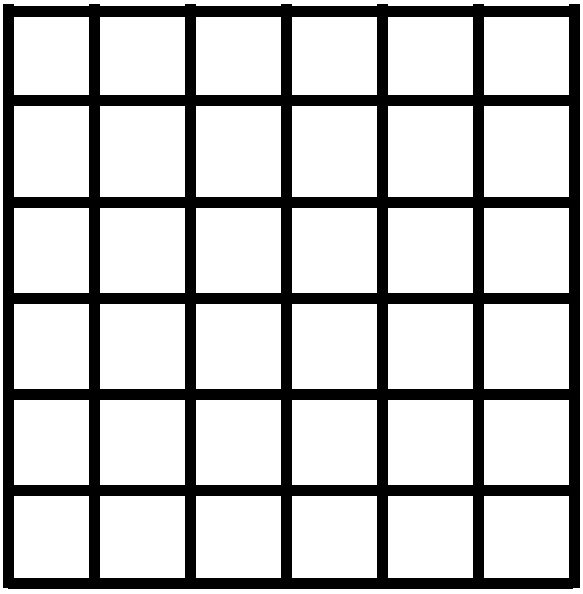
V = 25

Use the pieces shown to solve the following two puzzles:
(Note: There is more than one solution.)



Calculate the **Bottom and Top Area** for the two shapes shown:

Back side
A_B = _____



Left side
A_L = _____

Right side
A_R = _____

Top Area
A_{Top} = _____

Bottom Area
A_{Bottom} = _____

P = _____

Calculate the **Surface Area** for the two shapes shown:

SA = _____

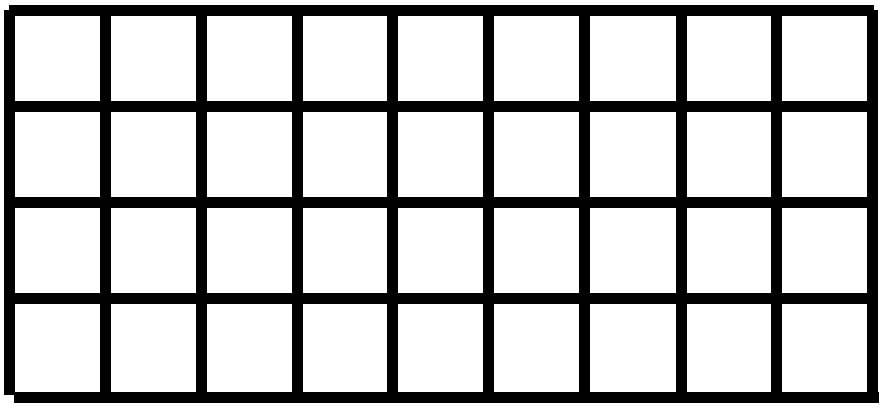
Front side
A_F = _____

V = _____

Calculate the **Parameter** of the two shapes shown:

Calculate the **Volume** for the two shapes shown:

Back side
A_B = _____



Left side
A_L = _____

Right side
A_R = _____

Top Area
A_{Top} = _____

Bottom Area
A_{Bottom} = _____

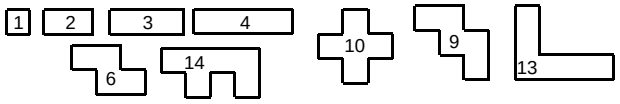
SA = _____

P = _____

Front side
A_F = _____

V = _____

Use the pieces shown to solve the following two puzzles:
(Note: There is more than one solution.)



Calculate the **Bottom and Top Area** for the two shapes shown:
Note the bottom and top areas are the same if the same pieces are used.

Back side
A_B = 6

6	4	4	4	4	2
6	6	3	3	3	2
13	6	9	9	1	14
13	9	9	10	14	14
13	9	10	10	10	14
13	13	13	10	14	14

Left side
A_L = 5

Right side
A_R = 6

Top Area
A_{Top} = 36

Bottom Area
A_{Bottom} = 36

P = 20

Front side
A_F = 6

SA = 96

V = 36

Calculate the **Side Areas** with the two shapes shown:
Note that the side areas are different even though the same pieces are used.

Calculate the **Surface Area** for the two shapes shown:
Note that the more compact the shape is the smaller the surface area.

Calculate the **Volume** for the two shapes shown:
Note that the volume didn't change

Calculate the **Parameter** of the two shapes shown:
Note that the parameter is different between the shapes.

Back side
A_B = 9

13	3	14	14	14	14	9	9	1
13	3	14	10	14	9	9	6	6
13	3	10	10	10	9	6	6	2
13	13	13	10	4	4	4	4	2

Left side
A_L = 4

Right side
A_R = 4

Top Area
A_{Top} = 36

Bottom Area
A_{Bottom} = 36

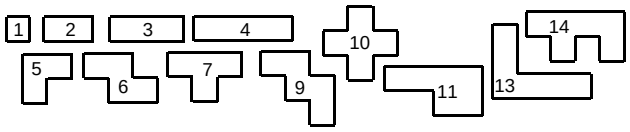
SA = 98

P = 26

Front side
A_F = 9

V = 36

Use the pieces shown to solve the following two puzzles:
(Note: There is more than one solution.)

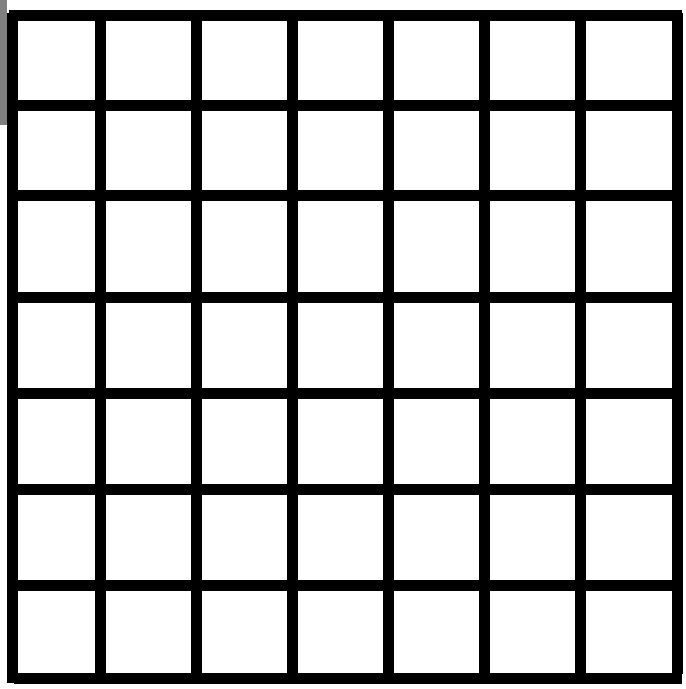


Calculate the **Bottom and Top Area** for the two shapes shown:

Back side
A_B = _____

Left side
A_L = _____

Top Area
A_{Top} = _____



Calculate the **Side Areas** with the two shapes shown:

Front side
A_F = _____

Right side
A_R = _____

Bottom Area
A_{Bottom} = _____

Calculate the **Surface Area** for the two shapes shown:

SA = _____

Calculate the **Parameter** of the two shapes shown:

V = _____

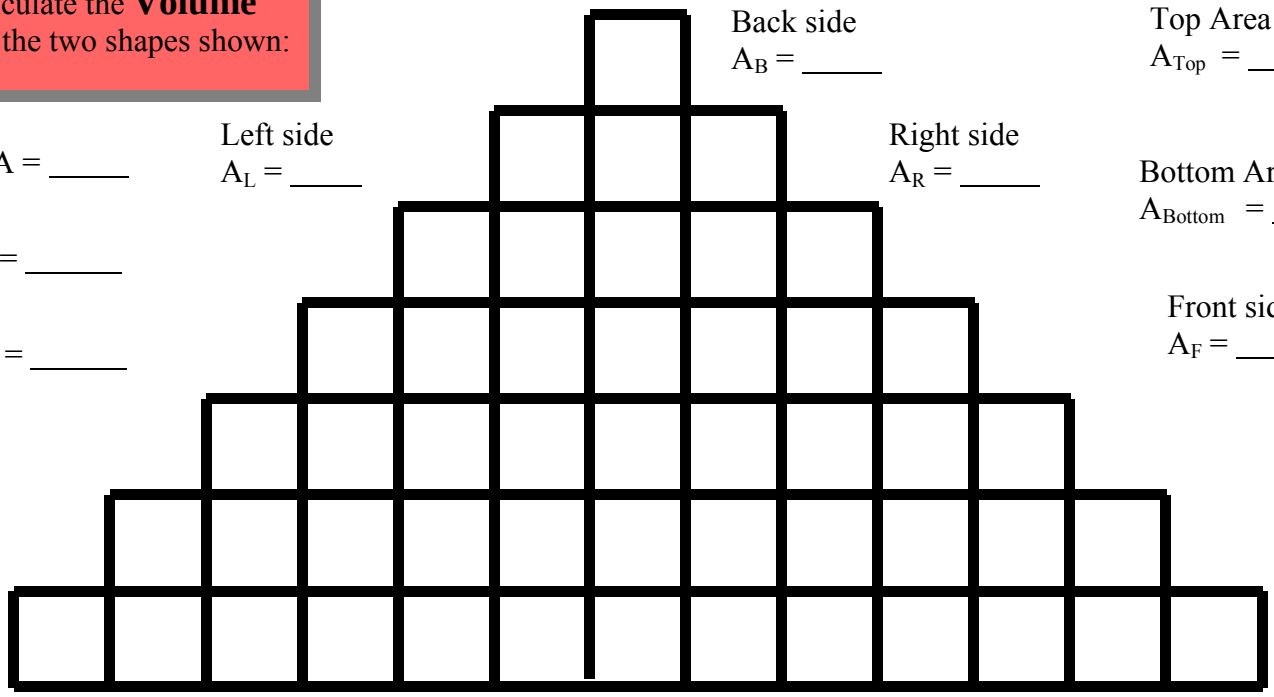
P = _____

Calculate the **Volume** for the two shapes shown:

SA = _____

P = _____

V = _____



Back side
A_B = _____

Left side
A_L = _____

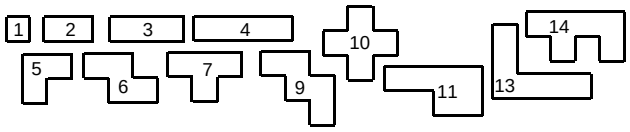
Right side
A_R = _____

Top Area
A_{Top} = _____

Bottom Area
A_{Bottom} = _____

Front side
A_F = _____

Use the pieces shown to solve the following two puzzles:
(Note: There is more than one solution.)



Calculate the **Bottom and Top Area** for the two shapes shown:

Back side
A_B = 7

Left side
A_L = 7

Top Area
A_{Top} = 49

7	4	4	4	4	1	2
7	7	6	3	3	3	2
7	6	6	9	9	11	11
13	6	9	9	10	11	11
13	5	9	10	10	10	11
13	5	5	14	10	14	11
13	13	13	14	14	14	14

Calculate the **Side Areas** with the two shapes shown:

Front side
A_F = 7

Right side
A_R = 7

Bottom Area
A_{Bottom} = 49

Calculate the **Surface Area** for the two shapes shown:

SA = 126

Calculate the **Parameter** of the two shapes shown:

P = 28

V = 49

Calculate the **Volume** for the two shapes shown:

SA = 138

P = 40

V = 49

Back side
A_B = 13

Left side
A_L = 7

Right side
A_R = 7

Top Area
A_{Top} = 49

Bottom Area
A_{Bottom} = 49

Front side
A_F = 13

					10						
				10	10	10					
			9	9	10	14	14				
		9	9	2	2	14	6	6			
	13	9	1	11	11	14	14	6	6		
	5	13	11	11	11	11	14	3	3	3	7
5	5	13	13	13	13	4	4	4	4	7	7	7

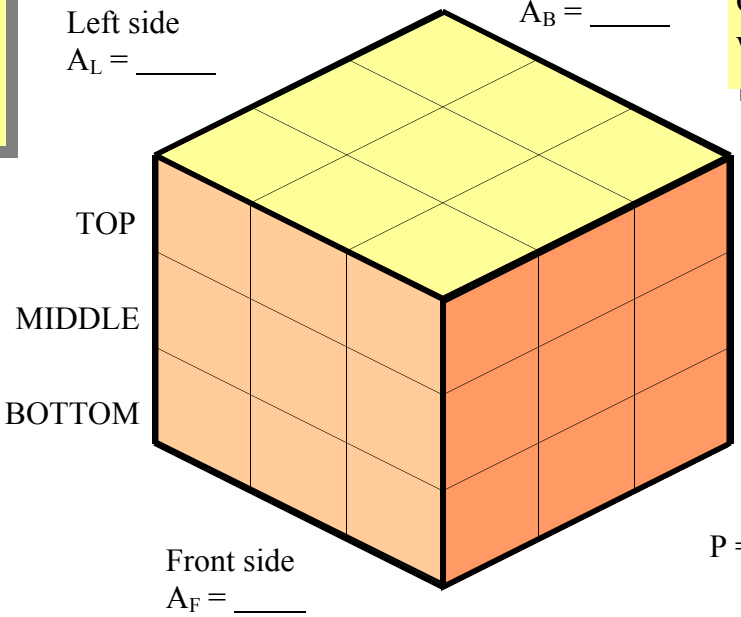
Calculate the **Bottom and Top Area** for the shape shown:

Top Area
A_{Top} = _____

SA = _____

Calculate the **Surface Area** for the shape shown:

Calculate the **Volume** for the shape shown:



Calculate the **Side Areas** with the shape shown:

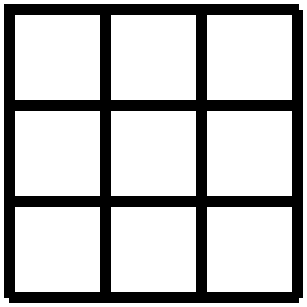
Right side
A_R = _____

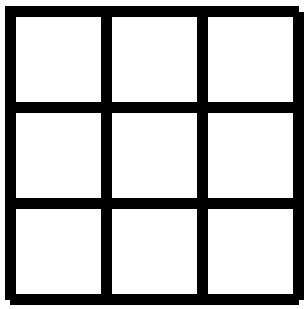
Bottom Area
A_{Bottom} = _____

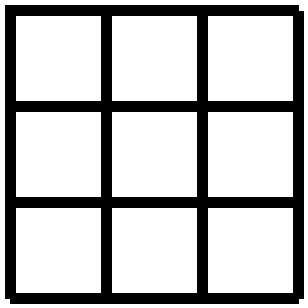
P = _____

V = _____

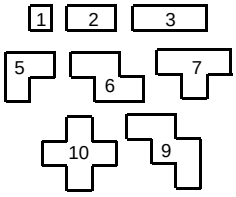
Calculate the **Parameter** of the shape shown:


TOP


MIDDLE


BOTTOM

Use the pieces shown below to make the 3x3x3 cub puzzle shown above. Record the piece locations in their respective



Calculate the **Bottom and Top Area** for the shape shown:

Top Area
A_{Top} = 9

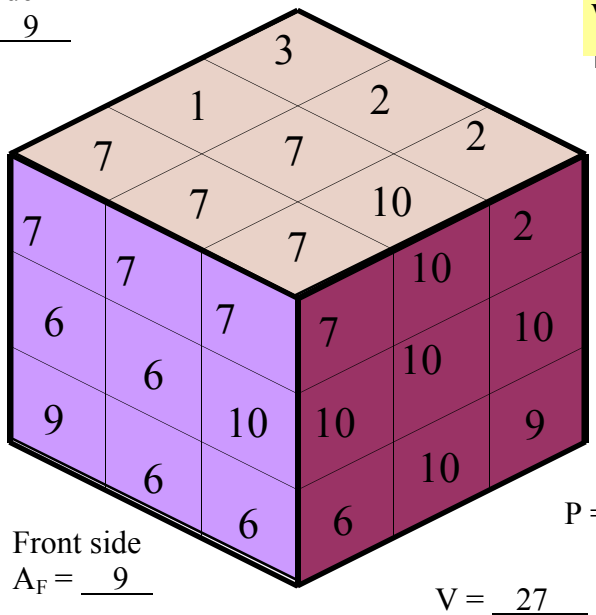
SA = 54

Calculate the **Surface Area** for the shape shown:

Calculate the **Volume** for the shape shown:

Left side
A_L = 9

Back side
A_B = 9



Front side
A_F = 9

Right side
A_R = 9

Bottom Area
A_{Bottom} = 9

P = 12

V = 27

Calculate the **Parameter** of the shape shown:

3	2	2
1	7	10
7	7	7

TOP

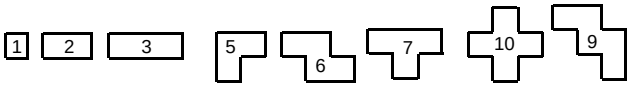
3	5	10
5	5	10
6	6	10

MIDDLE

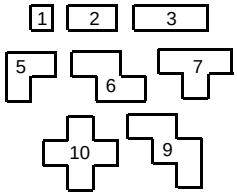
3	9	9
9	9	10
9	6	6

BOTTOM

Use the pieces shown below to make the 3x3x3 cub puzzle shown above. Record the piece locations in their respective



Use the pieces shown below to make the three different two-dimensional shapes shown on this page. Record the piece locations in their respective grid locations.



Back side
A_B = _____.

Top Area
A_{Top} = _____.

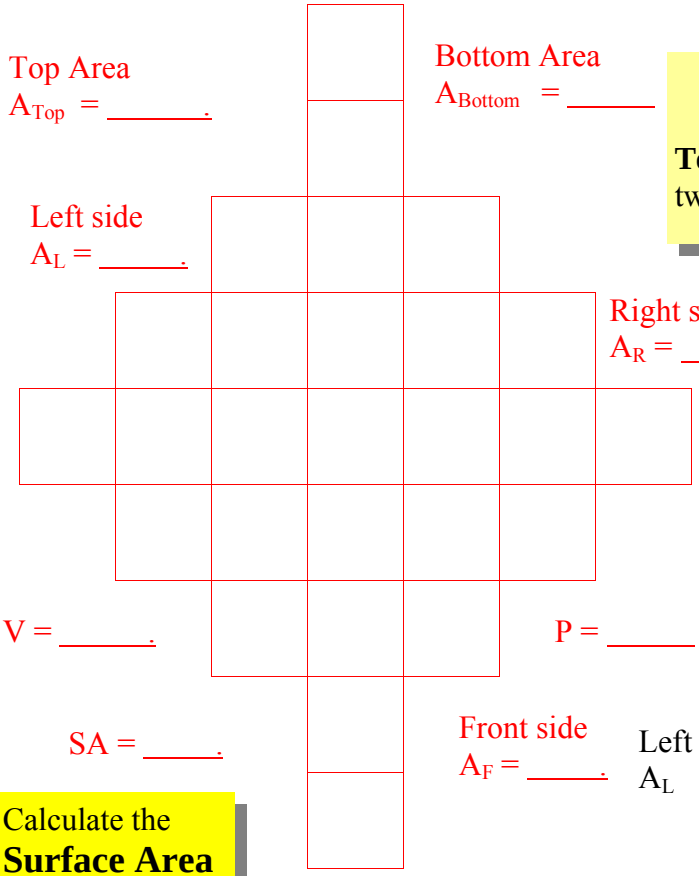
Bottom Area
A_{Bottom} = _____

Calculate the **Bottom and Top Area** for the two shapes shown:

Calculate the **Side Areas** with the two shapes shown:

Left side
A_L = _____.

Right side
A_R = _____.



V = _____.

P = _____

SA = _____.

Front side
A_F = _____.

Left side
A_L

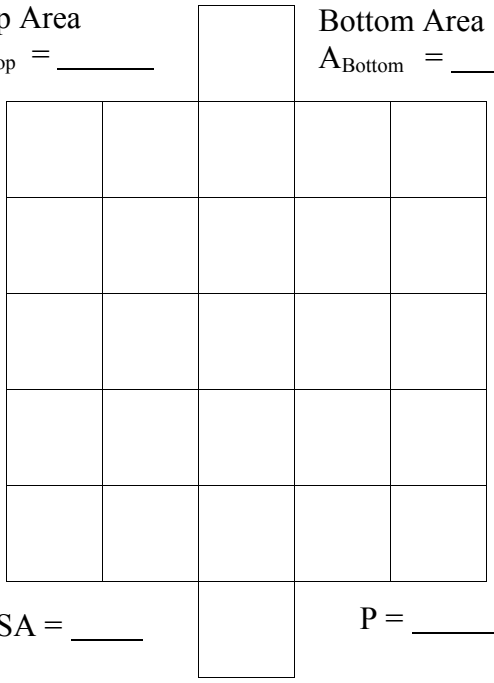
Calculate the **Surface Area** for the two shapes shown:

Calculate the **Parameter** of the two shapes shown:

Back side
A_B = _____

Top Area
A_{Top} = _____

Bottom Area
A_{Bottom} = _____



Right side
A_R = _____

V = _____

SA = _____

P = _____

Front side
A_F = _____

Calculate the **Volume** for the Three shapes

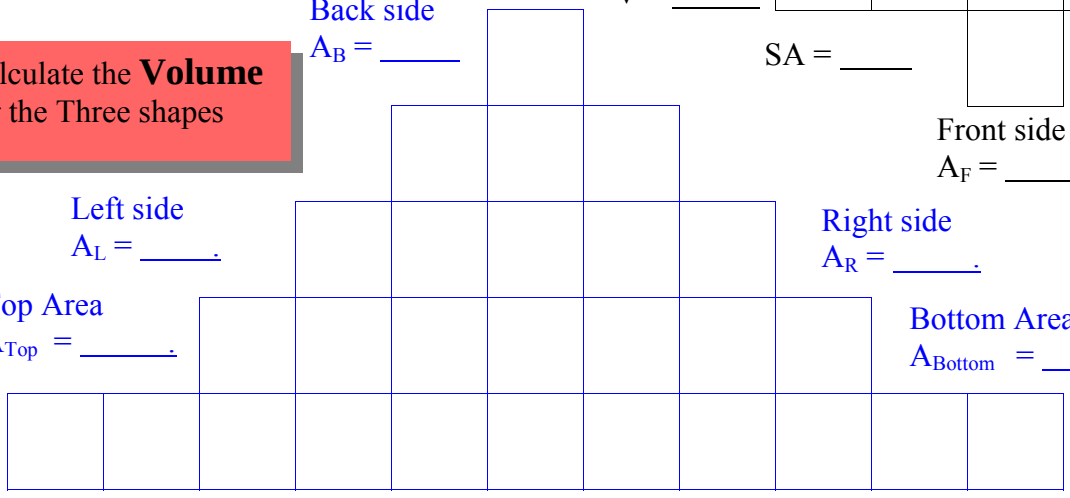
Back side
A_B = _____

Left side
A_L = _____.

Right side
A_R = _____.

Top Area
A_{Top} = _____.

Bottom Area
A_{Bottom} = _____.



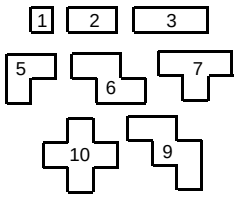
SA = _____.

Front side A_F = _____.

V = _____.

P = _____

Use the pieces shown below to make the three different two-dimensional shapes shown on this page. Record the piece locations in their respective grid locations.



Calculate the **Bottom and Top Area** for the two shapes shown:

Calculate the **Side Areas** with the two shapes shown:

Back side
A_B = 7

Top Area
A_{Top} = 27

Bottom Area
A_{Bottom} = 27

Left side
A_L = 9

Right side
A_R = 9

		3				
		3				
	6	3	9			
	10	6	6	9	9	
10	10	10	6	7	9	9
	10	5	5	7	7	
		5	2	7		
			2			
			1			

V = 27

P = 32

SA = 86

Front side
A_F = 7

Calculate the **Surface Area** for the two shapes shown:

Calculate the **Parameter** of the two shapes shown:

Back side
A_B = 5

Top Area
A_{Top} = 27

Bottom Area
A_{Bottom} = 27

		7		
1	7	7	7	3
5	5	2	2	3
5	9	9	6	3
9	9	10	6	6
9	10	10	10	6
		10		

Right side
A_R = 7

Left side
A_L = 7

V = 27

SA = 78

P = 24

Front side
A_F = 5

Calculate the **Volume** for the Three shapes

Back side
A_B = 11

Left side
A_L = 5

Right side
A_R = 5

Top Area
A_{Top} = 27

Bottom Area
A_{Bottom} = 27

				9						
			6	9	9					
		10	6	6	9	9				
	10	10	10	6	5	5	7			
3	3	3	10	2	2	5	7	7	7	1

SA = 86

Front side A_F = 11

V = 27

P = 32

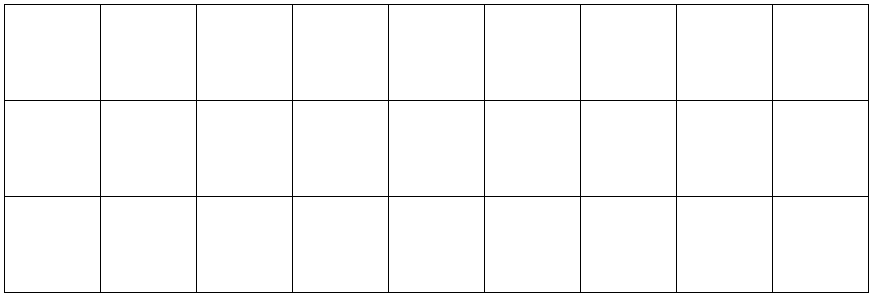
Top Area
A_{Top} = _____

Back side
A_B = _____

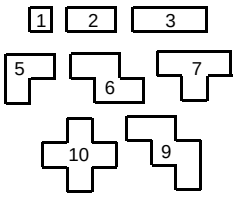
Right side
A_R = _____

SA = _____ P = _____

V = _____



Use the pieces shown below to make the two-dimensional and three-dimensional shapes shown on this page. Record the piece locations in their respective grid locations. (Note: There is more than one solution.)



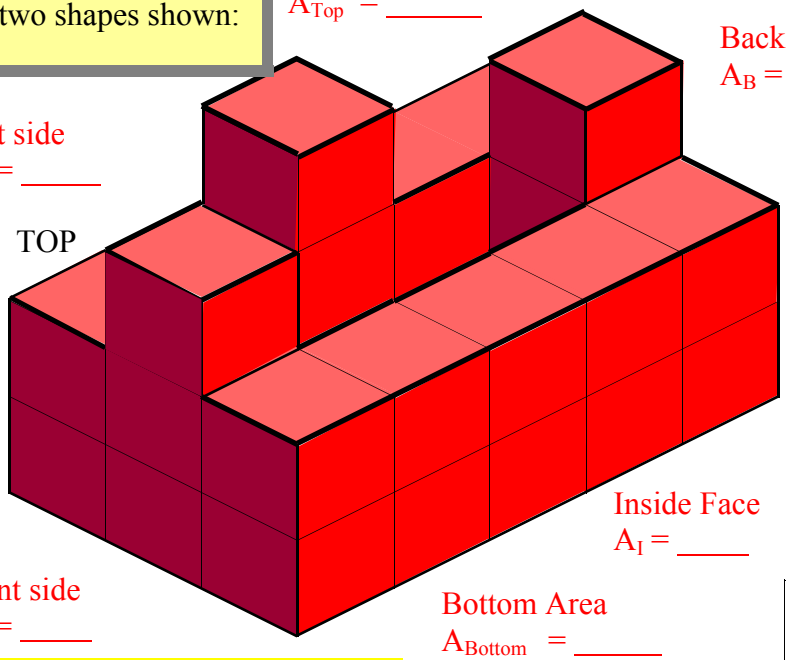
Left side
A_L = _____

Front side A_F = _____

Bottom Area
A_{Bottom}

Calculate the **Bottom and Top Area** for the two shapes shown:

Top Area
A_{Top} = _____



Back side
A_B = _____

Left side
A_L = _____

V = _____

SA = _____

P = _____

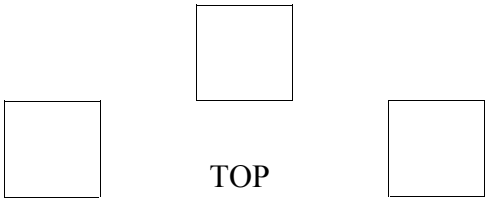
Calculate the **Side Areas** with the two shapes shown:

Right side
A_R = _____

Inside Face
A_I = _____

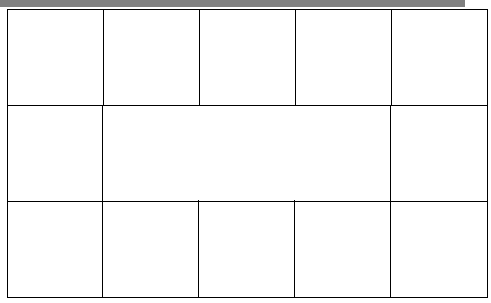
Front side
A_F = _____

Bottom Area
A_{Bottom} = _____



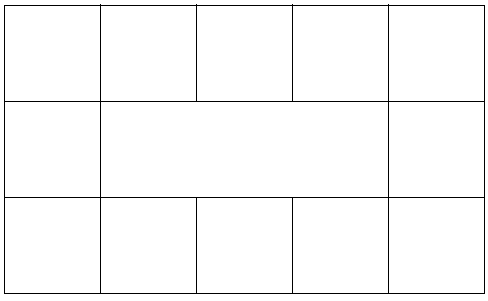
Calculate the **Surface Area** for the two shapes shown:

Calculate the **Parameter** of the two shapes shown:



MIDDLE

Calculate the **Volume** for the Three shapes shown:



BOTTOM

Top Area
A_{Top} = 27

Back side
A_B = 9

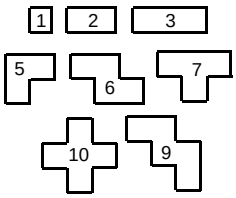
Right side
A_R = 3

SA = 78 P = 24

V = 27

5	5	3	3	3	10	9	9	1
5	6	6	7	10	10	10	9	9
6	6	7	7	7	10	2	2	9

Use the pieces shown below to make the two-dimensional and three-dimensional shapes shown on this page. Record the piece locations in their respective grid locations.



Left side
A_L = 3

Front side A_F = 9

Bottom Area
A_{Bottom} = 27

Calculate the **Bottom and Top Area** for the two shapes shown:

Top Area
A_{Top} = 12

Back side
A_B = 8

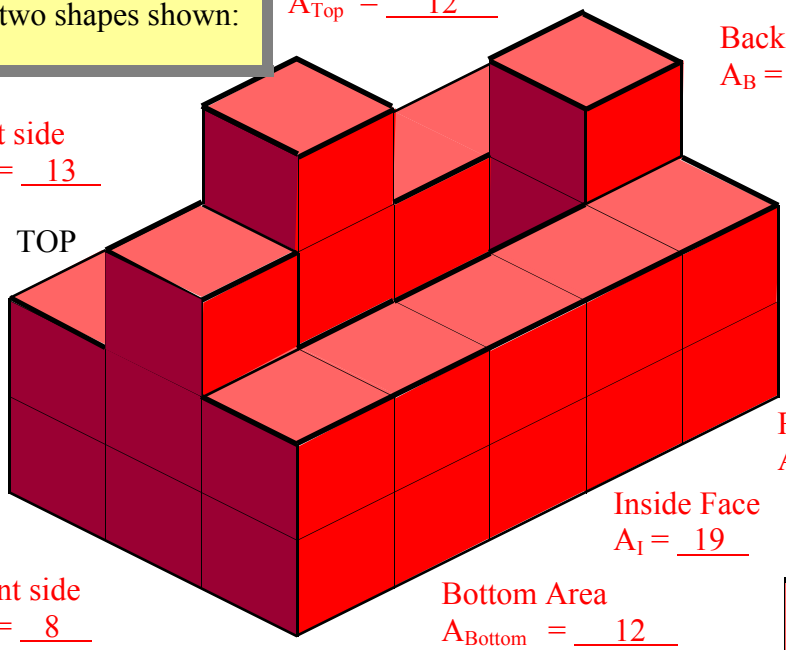
Left side
A_L = 13

V = 27

SA = 85

P = 16

Calculate the **Side Areas** with the two shapes shown:

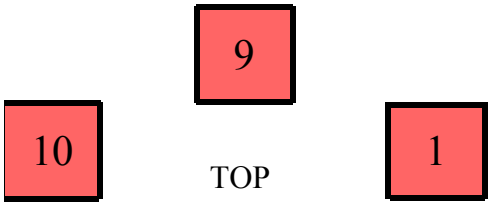


Right side
A_R = 13

Inside Face
A_I = 19

Front side
A_F = 8

Bottom Area
A_{Bottom} = 12



Calculate the **Surface Area** for the two shapes shown:

Calculate the **Parameter** of the two shapes shown:

Calculate the **Volume** for the Three shapes shown:

10	7	9	9	3
10				3
10	6	6	5	3

MIDDLE

7	7	7	9	9
10				2
6	6	5	5	2

BOTTOM

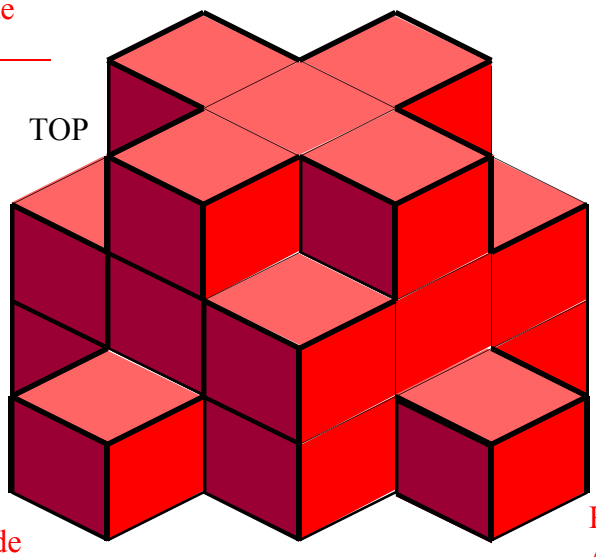
Top Area
A_{Top} = _____

Left side
A_L = _____

TOP

MIDDLE

BOTTOM



Back side
A_B = _____

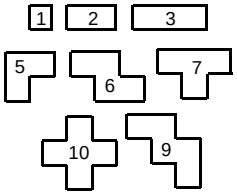
Right side
A_R = _____

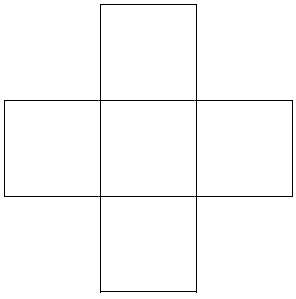
Front side
A_F = _____

Bottom Area
A_{Bottom} = _____

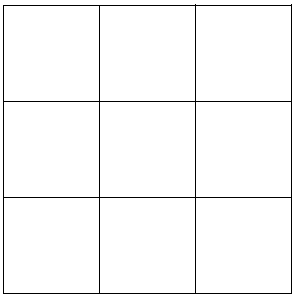
SA = _____ V = _____ P = _____

Use the pieces shown below to make the three-dimensional shape shown on the left. Record the piece locations at the bottom in their respective grid locations. (Note: There is more than one solution.) Then perform the calculations shown at the bottom of this page and record your answers around the shape accordingly.

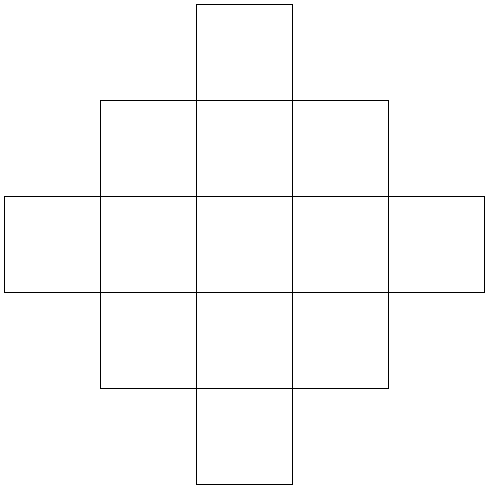




TOP



MIDDLE



BOTTOM

Calculate the **Bottom and Top Area.**

Calculate the **Side Areas.**

Calculate the **Surface Area.**

Calculate the **Parameter.**

Calculate the **Volume.**

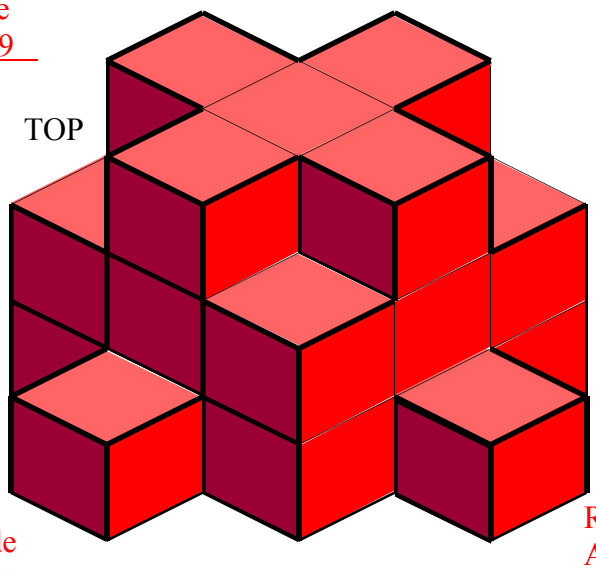
Top Area
A_{Top} = 13

Left side
A_L = 9

TOP

MIDDLE

BOTTOM



Back side
A_B = 9

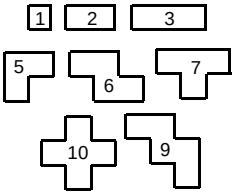
Right side
A_R = 9

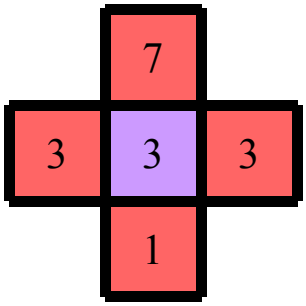
Front side
A_F = 9

Bottom Area
A_{Bottom} = 13

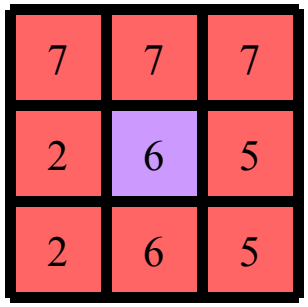
SA = 62 V = 27 P = 20

Use the pieces shown below to make the three-dimensional shape shown on the left. Record the piece locations at the bottom in their respective grid locations. (Note: There is more than one solution.) Then perform the calculations shown at the bottom of this page and record your answers around the shape accordingly.

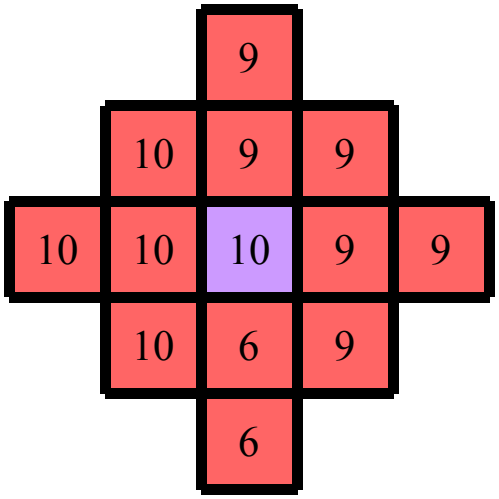




TOP



MIDDLE



BOTTOM

Calculate the **Bottom and Top Area.**

Calculate the **Side Areas.**

Calculate the **Surface Area.**

Calculate the **Parameter.**

Calculate the **Volume.**